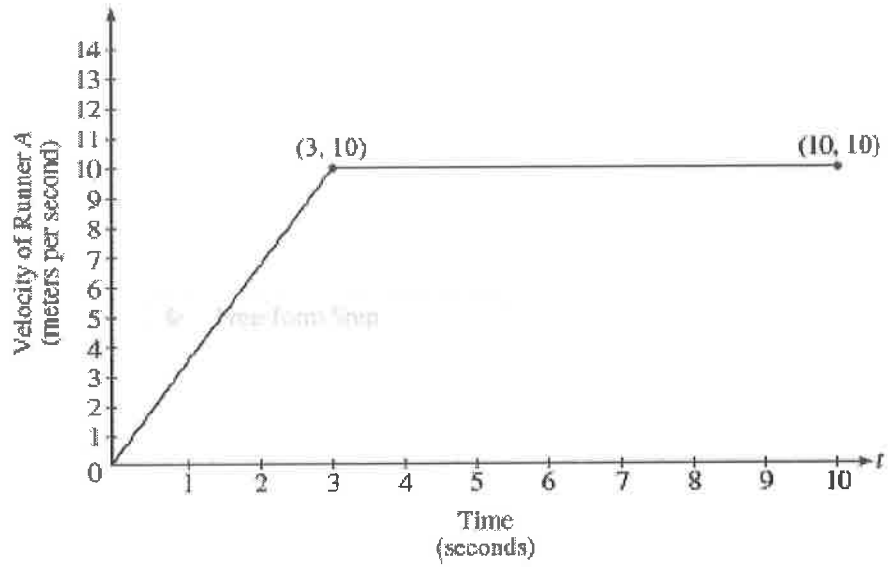
Key

7-1 Integral as Net Change Practice

1. AP 2000-2



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

Runner A

For $0 \le t < 3$

$v(t) = \frac{10}{3}t + 0$

$v(2) = 6\frac{2}{3}$ m/s

Runner B

$v(2) = \dfrac{24(2)}{2(2)+3} = 6\frac{6}{7}$ m/s

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

Runner A

$a(2) = \frac{10}{3}$ m/s^2

Runner B

$a(2) = v'(2) = 1.469$ m/s^2

(from calculator)

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

Runner A

one big trapezoid

$D = \frac{1}{2}(10+7)10$

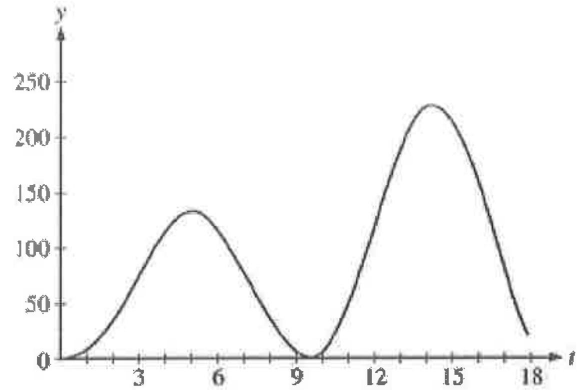
$= 85$ meters

Runner B

$D = \int_0^{10} v(t)\,dt$

$= 83.336$ meters

2. AP 2006-2



At an intersection in Thomasville, Oregon, cars turn left at the rate of $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

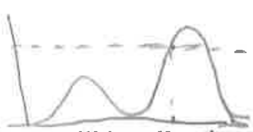
$\int_0^{18} L(t)\,dt = 1657.82 \approx 1658$ cars

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

$L(t) \ge 150$

From graph:

for $12.428 < t < 16.122$
$\quad\quad A \quad\quad\quad\quad B$



$\text{Ave}(L) = \frac{1}{B-A}\int_A^B L(t)\,dt$

$= 199.417$ cars per hour

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

Max number of cars turning left happens at $t = 14.214$ hrs.

Look at $500 \cdot \int_{13.214}^{15.214} L(t)\,dt$

$= 217,790$ which exceeds the 200,000 required.

Put in that light!

3. AP 2008-2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

$$L'(5.5) \approx \frac{L(7) - L(4)}{7-4} = \frac{150-126}{3} = 8 \text{ People per hour}$$

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

$$\text{Ave} = \frac{1}{4}\int_0^4 L(t)\,dt = \frac{1}{4}\left[\left(\tfrac{1}{2}\right)\left(1(156+120) + 2(176+156) + 1(126+176)\right)\right]$$

$$= 155.25 \text{ People}$$

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer. $L(t)$ is twice differentiable, $L(t)$ and $L'(t)$ continuous MVT and IVT apply. MVT implies $L'(t) >$ for some $t \in (1,3)$ and some $t \in (4,7)$. Also $L'(t) < 0$ for some $t \in (7,8)$, Since L' is continuous, IVT implies $L'(t) = 0$ at least 3 times.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number.

$$\text{Tickets sold} = \int_0^3 r(t)\,dt = 972.784 \approx 973 \text{ tickets}$$